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Gravitational Waves

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Abstract

Newton's theory of gravity does not allow for gravitational waves but consistency demands that they be there. Einstein's general theory of relativity requires that there be gravitational waves. There is already indirect evidence that gravitational waves exist and there are tremendous efforts being made to try to get direct evidence for them. Here the subject of gravitational waves will be reviewed, starting with the need of consistency of classical mechanics and then going on to explaining why they must be there and the complications arising in relativity from their existence and the attempts being made to observe them.

Key Words: Gravitational Waves, Theory of General Relativity& Gravitation

1. Introduction

Newton's theory of gravitation, which he called a "law" of gravitation, is inconsistent. How so? I will explain. The problem arises with his world-view. He posits an absolute space, to be visualized as a rigid 3-dimensional grid and an absolute time, to be represented by a vertical rigid line going up and to be regarded in some sense as "orthogonal" to the grid. "Ponderous" matter, conceived of as consisting of particles, occupies is present at some point in the grid and possesses a property of resisting changes in its motion, which is called inertia and measured by a "mass". Newton's theory of gravity then requires that the masses *instantaneously* attract each other with a force that is proportional to each of the masses and to the inverse square distance between them. There is no mechanism for one particle to attract the other – it just does so. The problem of consistency is not the lack of mechanism, thought it may well bother one, it is the instantaneous action at a distance. Another way os stating the problem is that the theory requires the gravitational field to be *static*. As such, it cannot accommodate moving (and especially accelerating) masses. “Come on", you might say, “ *of course* it incorporates moving masses. How else would it provide for the motion of planets about the Sun? In fact it gives precise formulae for how the masses will move." I will explain my claim.

The gravitational potential is just $\phi(r) = -GMm/r$, which does not have any time dependence in it. The way it handles this problem is to allow the distance between the particles to depend on time, so that r becomes $r(t)$. Thus the gravitational force, $F = -\nabla\phi$ will be time dependent. So why is that inconsistent? The problem is that the effect of the change is *instantaneous*. This may not seem like a problem with an absolute time, as one then can get knowledge of what happened somewhere else “at the same time". There isanother related problem. The changing force acting over some distance will provide some work done by the field. In other words there is energy radiated. Further, the scalar product of the velocity of the particle with the force will give the power radiated. *But what is the nature of this energy radiated?* Neither of these problems can be said to prove inconsistency. However, the power generated must come from the time derivative of the gravitational potential energy and is related to the second derivative of the position. Thus we get a D’Alembert equation. The D’Alembert equation requires a speed for the disturbance but that is not allowed! *This* is the insurmountable problem. In effect Newton’s gravitational field is static. We need a dynamic field. The resolution of these problems is provided by Einstein’s general theory of relativity.

When Coulomb formulated his “law" for the electric force between two charges, he modeled it on Newton’s law of gravity. Its later extension to the magnetic force was again modeled in the same way, despite the fact that it is only in a very odd way that the two could be treated as the same. After all, masses and charges can be approximated as points but there are no magnetic monopoles seen. By the further approximation of taking two very long bar magnets with the two poles close to each other, can the Coulomb force fit the Newtonian model. Why was gravity taken as a model for the electric and magnetic forces? The reason is not scientific but sociopolitical. Newton had been touted as ‘the greatest scientist of all time" and having “discovered the laws of Nature" by the English. While Europe did not so readily agree to accept that status for Newton, the sheer hype of the British carried Newton onto a pedestal and subsequently distorted the development of Science. All wanted a “mechanical model" of physical reality. The fact is that the idealization for electric charges happens to be very good. However, the study of electricity and magnetism showed that their behaviour was totally different from that of gravity. Many scientists wasted a great deal of effort in trying to force the newly observed phenomena into the mechanical mould. Now magnets have a remarkable property that can be seen easily and leads to a very different visualization from that entailed in gravity. When a bar magnet is placed on a sheet of paper and iron filings are spread about it uniformly, they show patterns that clearly depict the lines of force about the magnet. Faraday treated these lines of force as very real and describing a field of force about the magnet. This concept of a field changes the way one thinks of Physics. It came into its own with the work of James Clerk Maxwell. He spent a lot of time and effort to construct mechanical models for the phenomena so well explained by his theory. He demonstrated that it could be done but the cumbersome models actually seemed to show why it *should not* be done. Why not take electromagnetism as a model for gravity rather than gravity as a model for electromagnetism. (A discussion of Maxwell’s contributions in given in (Qadir and Mason, 2015)

The start of the solution comes from Maxwell. His unified theory of electricity and magnetism, or *electromagnetism*, forced a “displacement current" into Ampere’s empirical law relating currents to magnetic effects. Maxwell checked that this current was physically present. With this term in place the Maxwell’s equations imply that the electric and magnetic fields satisfy the D’Alembert equation, with the speed of the electromagnetic disturbance in a medium, $c_m = 1/\sqrt{\varepsilon\mu}$, where ε is the dielectric constant and μ the magnetic permeability of the medium. What happens to the speed for a vacuum? Well, we can measure the dielectric constant and magnetic permeability of the vacuum, ε_o, μ_o in the laboratory and use it to obtain the value of the speed defined. It turns out to be the same as the speed of light in avacuum. Using the values in other media and taking the ratios of the speeds to the vacuum value, we find that the ratios fit precisely with the refractive index. Thus we can identify light as the electromagnetic disturbance. D’Alembert’s equation is a wave equation, so we can conclude that light travels as a wave, in that it satisfies a wave equation. The speed obtained here does not depend in any way on the state of motion of the source of the wave or its observer. Automatically, Maxwell’s theory says that the speed of light depends on the nature of the medium and not the state of motion of sources and observers. Since the dielectric constant and magnetic permeability are least for the vacuum, the speed of light in the vacuum is the maximum speed for an electromagnetic disturbance. Further, scientists before Maxwell, he himself and many who followed were convinced by themechanical model for waves that there must be a medium for electromagnetic waves, which was called "the luminiferous aether". However, his theory dispenses with the aether as the electric and magnetic fields provide the "medium" in themselves. One wonders why Maxwell, despite his superb physical insight, did not see so many of the obvious implications of his own theory. The only answer seems to me the unhealthy influence of Newton reaching out of the grave and out of time.

2. Brief Review of Special and General Relativity

It took Einstein to see the obvious implications of Maxwell's theory. He did not subscribe to "Newton worship" and regarded Maxwell as being at par with Newton. Other people were getting there but without the clarity that Einstein achieved because he was not wedded to Newton's world-view. In 1905 he dispensed with the assumptions of an absolute space and an absolute time, allowing them to be determined by a thought experiment. He considered two light signals sent in opposite directions by two observers when at the time that they coincided and in the direction of the relative motion. He also made explicit the prevalent assumption that it is impossible to determine which of two relatively non-accelerated observers is moving and which is at rest, without reference to a third observer. Allowing the two observers to give the position and time for the signal front from the time and position when it was sent, in their own separate ways he could give the transformation of the descriptions from one to the other. This resolved a problem that Lorentz had faced when he tried to use Maxwell's theory for point charged particles. With that he also showed that lengths of moving bodies would appear to be contracted along the direction of motion. This had been proposed separately by Fitzgerald and Lorentz to explain the null results of Michelson and Morley, when they tried to measure the motion of the Earth through the luminiferous aether. Of course, there was no aether relative to which the Earth would be moving in the first place. Along with the length contraction there was a dilation of time predicted. This has since been confirmed by direct observation, by keeping one atomic clock in the laboratory and taking another seven times around the Earth in a jet plane. The time difference was noted and then the atomic clocks were switched and the experiment done again. When the slow clock was found to have caught up with the faster clock on reversal, it confirmed the prediction fully.

One might ask why *light* should be used to send the signals. The natural answer would be that it is the fastest way to send a signal. The first caveat is that light does not move faster than electrons (say) in a medium. Well, we would then use the light signals sent in vacuum. But the problem is that they may be the fastest signals *we* know of, but an advanced civilization may know of faster signals.

Would Einstein's theory then be wrong? As explained in (Qadir, 1989) one could replace light by the fastest possible signal. You might object that this is not valid. What if there is no maximum speed? Well, let us derive results assuming that there is some limit, determine the value of that limit and then test the resulting theory. Doing this, as explained above, we would obtain a value for the maximum speed which would be approximately the speed of light in vacuum. Of course, it could not be less in any case but it could be more. By the observation very stringent limits on the excess would appear. This would come from length contraction. Then, time dilation would provide the desired test. Thus light happens to travel at (or nearly at) the maximum possible speed and it is not that wehave to assume that light is the fastest signal. Separately, from Maxwell's theory, we know that light must travel as a speed that is constant for all observers. Practically all of the special theory of relativity is contained in this discussion. What remains was provided by Einstein in 1908, when he deduced a formula for the mass appears to change with relative motion and the consequent equivalence of mass and energy. The argument used by Einstein bounced two billiard balls off each other, thereby introducing a change of the frame by acceleration. This is not an error but is not satisfying for a theory based on non-accelerated motion. One can use the elegant formalism of Hamilton to obtain both results without appealing to any acceleration, as explained in the above reference.

The Lorentz transformations, written in differential form, automatically give formulae for the relativistic addition of velocities, which leave the speed of light invariant. Starting with mathematical reasoning, Poincaré had arrived at the velocity addition formulae about the same time as Einstein and always felt that credit for discovery of the theory should have gone to him. However, he did not, himself, believe any of the physical consequences of the theory and so his claim rings hollow. Similarly, the moving mass formulae was seen as required in some experiments with beta rays had been seen by Kaufmann. However, once again the "law of the conservation of mass" stood in th way of accepting the variation. It is the clear understanding of the two phenomena arising from some extremely simple considerations that gives Einstein the credit for constructing the special theory of relativity. He went further some time around 1911 or so and conceived a thought experiment for uniform *circular* motion. This, of course, is accelerated motion. He noted that an observer sitting at the edge of a disk of radius a rotating with an angular speed ω, would see a small rod of length l lying tangential to the disc as shrunk due to Lorentz contraction, by a factor of $\gamma = \sqrt{1-\omega^2 a^2/c^2}$ but would see the diameter, d as unchanged, since the rod would lie along the direction of motion and the diameter perpendicular to it. Now if n of these rods make up the circumference $nl/d = \pi$ if the disc is not rotating. Thus the ratio of the circumference to the diameter would be $\gamma\pi$. By definition, the ratio of the circumference to the diameter in Euclidean geometry is π. Thus, the thought experiment demonstrated that the geometry could not be Euclidean! And this is for one of the the simplest forms of accelerated motion.

When Einstein had first formulated his special theory of relativity, his one-time Mathematics teacher (whose classes Einstein had often missed) re-wrote the theory in terms of a 4-dimensional geometry. Einstein had been strongly opposed to this development because, he felt, it distracted from the physics to a mere mathematical formalism. However, once he realized that he needed to convert from the natural intuition based on Euclidean geometry, he started trying to learn about non-Euclidean geometries. He tried affine geometry and teleparallel geometry but they did not provide a physical formulation that Einstein found at all satisfactory. Finally he went to his mathematician friend Marcel Grossmann, who had taken Minkowski's lectures much more seriously to learn about Riemannian geometry. This led him to collaborate with Grossmann in differential geometry and to apply it for the extension of his theory to accelerated motion. Thus from 1911 to 1915 Einstein tried to formulate many theories and only in 1915, a decade after his first paper on Relativity, did he resolve the problem.

The main hurdle to be resolved as far as the physics was concerned was that while one cannot feel non-accelerated motion locally, one does feel acceleration. How, then, can one generalize theprinciple of special relativity that all inertial frames are equivalent. I already provided a minor extension by saying that all relatively non-accelerated are equivalent, but that would hardly extend the theory to accelerated motion.. Einstein started with the realization that mass plays a dual role in Physics.On the one hand it gives the inertia, or resistance to change, of Newton's second law of motion. On the other hand it acts as a *gravitational charge*, analogous to the electric charge. There seemed to be no good reason for this identity. In fact, Count Eötvos had experimentally verified that the masses measured in both ways are equal. Einstein took this as a fundamental principle of his general theory under the title "principle of equivalence". This was the same thing that Einstein had checked at the leaning tower of Pisa, when he dropped two stones of different masses together and they fell together. He noted that if Galileo had jumped from the tower along with the stone he would see the stone as being unaffected by the gravitational force. In other words one could "switch off" gravity by going into free-fall. While the description would change from one frame to another, the physical laws would have to be the same and should describe the same geometric motion. What one observer may see as a straight line another may see as a curve. He called this the "principle of general covariance". The original purpose had been to provide a theory of arbitrary motion but he had not got a handle on all forces, only on the force of gravity. As such, his first step was to develop the theory of arbitrary motion under gravity. Instead of taking only inertial frames he could now take all frames in which there are no forces other than gravity. Gravity had been rendered harmless by the principle of equivalence.

Having effectively got rid of Newton's second law, we only need the generalization of Newton's first law, which states that bodies continue in their state of rest or of uniform motion unless an external force acts on them. Since there are no external forces and there is no difference between being at rest or in uniform (linear) motion, bodies should continue in a straight line. However, what one observer saw as a straight line another would see as a curved path. Thus we need to deal with curved spaces where there *is* no straight line, only "straightest *available* paths", called *geodesics*. Particles should, then, move along geodesics in the curved spacetime. (Following John Wheeler, we write the two words together so as to emphasize the unification of space and time into a single entity).

How would the curvature come into mechanics? Recall the local measure of (arc) length, ds, called the *metric* and is given by the *metric tensor*, $g_{\mu\nu}$, by $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$, where dx^{μ} is the infinitesimal displacement vector and $\mu = 0,..,3$, x^0 being ct, where c is the speed of light and t is the coordinate time and dx^i is the spatial vector $(i = 1,2,3)$. Remember that the arc length parameter in relativity is $ds = cd\tau$, where τ is the proper time. Thus a derivative of the position vector with respect to it gives the 4-velocity, u^{μ}, and the second derivative gives the acceleration A^{μ}. Thus if an observer is moving without any force the path should be the straightest available path in the spacetime. It should not change direction. Thus the directional derivative of the unit tangent vector along the path should be zero. The directional derivative of any vector V^{μ} along t^{μ} is given by

$$dV^{\mu}/ds := t^{\nu}V^{\mu}_{;\nu} = t^{\nu}V^{\mu}_{,\nu} + t^{\nu}\Gamma^{\mu}_{\nu\rho}V^{\rho}, \qquad (1)$$

where the semicolon represents the covariant derivative, the coma a partial derivative with respect to the position vector x^{μ} and $\Gamma^{\mu}_{\nu\rho}$ is the Christoffel symbol, given by

$$\Gamma^{\mu}_{\nu\rho} = g^{\mu\sigma}(g_{\rho\sigma,\nu} + g_{\nu\sigma,\rho} - g_{\nu\rho,\sigma}), \qquad (2)$$

$g^{\mu\sigma}$ being the inverse metric tensor, i.e. $g^{\mu\sigma}g_{\nu\sigma}=\delta^{\mu}_{\nu}$, and the coma denoting partial differentiation with respect to the position 4-vector and using the Einstein summation convention that repeated indices are summed over the entire range of indices (in our case $0,...,3$). The Christoffel symbol is not a tensor, being coordinate dependent. In a flat space we can use Cartesian coordinates and make it zero. However, in a curved space there would *be* no Cartesian coordinates and it could not be made zero in any region of the space but only at one point. The condition for the straightest available path, $dt^{\mu}/ds=0$, then becomes

$$\ddot{x}^{\mu}+\Gamma^{\mu}_{\nu\rho}\dot{x}^{\nu}\dot{x}^{\rho}=0, \tag{3}$$

as $t^{\mu}=\dot{x}^{\mu}$. Such a path is called a *geodesic* and this equation is called the *geodesic equation*. Clearly, the Christoffel symbol is a linear combination of the first derivatives of the metric tensor. The curvature should be given by the *second* derivative. Riemann's generalization of Gauss' theory for curved surfaces to n-dimensions gives the curvature tensor as

$$R^{\mu}_{\nu\rho\pi}=\Gamma^{\mu}_{\nu\pi,\rho}-\Gamma^{\mu}_{\nu\rho,\pi}+\Gamma^{\mu}_{\sigma\rho}\Gamma^{\sigma}_{\nu\pi}-\Gamma^{\mu}_{\sigma\pi}\Gamma^{\sigma}_{\nu\rho}. \tag{4}$$

Now representing the position vector of the particle being observed, as seen by the observer, by p^{μ}, the acceleration vector must be the second directional derivative of this vector along the observer's geodesic. Thus $A^{\mu}=d^{2}p^{\mu}/ds^{2}$. It is called the geodesic deviation. Evaluated in terms of the Christoffel symbols it surprisingly yields

$$A^{\mu}=R^{\mu}_{\nu\rho\pi}t^{\nu}p^{\rho}t^{\pi}. \tag{5}$$

This provides the connection between the curvature of spacetime and gravity and so can get rid of gravity and Newton's second law in one stroke, by reducing both to the curvature of the spacetime. We now need to express the distribution of matter quantitatively. It would seem that the distribution of matter should simply be the mass density, ρ. However, from special relativity we learn that mass and energy are equivalent and that we cannot talk of energy and momentum separately but must deal with the energy-momentum 4-vector. Further, one not only deals with kinetic energy but also potential energy. The potential energy stored may simply be as a scalar measure of the energy, but may also be contained in stresses in the material, be it solid or fluid. A (2^{nd} rank) stress tensor is needed for the mechanics of continuous media. Thus we need a 4-dimensional tensor that contains both the stress tensor and the energy-momentum vector. Obviously we cannot add a vector to a second rank tensor and so would like to take two copies of the energy-momentum 4-vector together to provide a 2^{nd} rank tensor. However, the resulting quantity would not have the units of energy density. The energy-momentum vector is just the mass density times the velocity 4-vector, t^{μ}. Thus the energy-momentum density vector would be the mass density times the 4-velocity. Now one can take two copies of the velocity 4-vector times one copy of the mass density. Writing the stress tensor as σ^{ij} we run into another problem. How do we add a 4-d tensor to a 3-d tensor? For this purpose go to the rest-frame of the mass-energy distribution and now write the tensor $S^{\mu\nu}$ to be the continuum stress-tensor for the spatial index values and zero otherwise. Then the relativistic stress-energy tensor density would be $T^{\mu\nu} = \rho t^{\mu} t^{\nu} + S^{\mu\nu}$. Now one can go to any other frame by applying Lorentz transformations to this tensor. In the rest-frame t^{μ} has only the time component and the space component is zero by definition.However, one cannot simply put the time component to be c, as $g_{\mu\nu} t^{\mu} t^{\nu} = c^2$. Thus $t^0 = 1/\sqrt{g_{00}}$. For an isotropic perfect fluid with pressure p,

$$T^{\mu\nu} = (\rho + p/c^2) t^{\mu} t^{\nu} - p\delta^{\mu}_{\nu}, \tag{6}$$

Our next problem is that we cannot easily relate the 2^{nd} rank stress-energy tensor to the 4^{th} rank curvature tensor. To get the same rank we take the trace of the Riemann tensor, called the *Ricci* tensor, $R_{\nu\pi} = R^{\mu}_{\nu\mu\pi}$. Further, the stress-energy tensor must be conserved. By Gauss' divergence theorem, that means that its divergence has to be zero. The curvature tensor satisfies the differential Bianchi identities, which on two contractions yield the requirement that

$$(\varepsilon^{\mu}_{\nu})_{;\mu} := (R^{\mu}_{\nu} - \frac{1}{2} R\delta^{\mu}_{\nu})_{;\mu} = 0, \tag{7}$$

where R is the trace of the Ricci tensor, called the Ricci scalar. Einstein assumed the simplest possible relation between the two, namely a linear one:

$$\varepsilon^{\mu}_{\nu} = \kappa T^{\mu}_{\nu} - 2\Lambda\delta^{\mu}_{\nu}, \tag{8}$$

where κ is a constant of proportionality and Λ a constant of integration. These are known as the Einstein field equations.

For general relativity (GR), then, Einstein assumed: (a) the principle of general covariance, that physical laws should be expressible in tensorial form; (b) the principle of equivalence, that gravitational and inertial masses are equivalent; (c) the relativistic "laws of motion" that test particles will move along the straightest available paths (geodesics); (d) Mach's principle (which he later dropped) that the frame of zero acceleration is that of "the distant stars"; and (e) that special relativity will apply locally, i.e. at sufficiently small scales. The last is going to be of particular relevance to us. Geometrically it is the same as saying that the tangent plane locally approximates the curved space to which it is tangent. This is trivially obvious but is significant. I will discuss this point later.

3. Gravitational Waves in Relativity

In GR the potential is a tensor, but that is not the critical difference from Newtonian gravity or electrodynamics. The field equations in these cases are linear (partial) differential equations. However, in GR we get products of the Christoffel symbols, which are themselves nonlinear functions of the field, namely the metric tensor, as they involve the inverse matrix as well as derivatives of the matrix. This nonlinearity destroys the possibility of being able to isolate causes in that the initial conditions in one spacetime region can make an unlimited difference in some other spacetime region. As such, the full theory really requires that one describes the whole Universe, gives the initial conditions everywhere in it and then determines the state of the Universe from there on, solving the field equations. In this picture the history of the whole Universe is "written-in" from the start and then nothing happens outside that history, There are no "test-particles" that were not written in at the start — all we have is "a painted ship upon a painted ocean". This is why Einstein decided to apply his theory to the Universe as a whole and so founded the science of Cosmology. To be able to apply the theory to actual problems, like the bending of light or the perihelion shift of Mercury (the big successes of GR) one has to use an approximate version of the theory in some way. One way is to approximate the equations by linear ones, solve them, use the solution to approximate the nonlinear terms and now solve the modified linear equations. Usediteratively, it would appear that one could get arbitrarily close to the true nonlinear theory. This method is used for the above-mentioned problems. Instead, we can just take the linear approximation and try to coax all the insights of the nonlinear theory from it.

For the linearized theory of gravity we write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ is a perturbation on it. Since $g_{\mu\rho} g^{\nu\rho} = \delta^{\nu}_{\mu}$, we easily see that to lowest order $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ Then, for the Christoffel symbols the derivative will only be of $h_{\mu\nu}$ and the inverse metric tensor will be approximated by $\eta^{\mu\nu}$. Hence this tensor will be used to raise or lower indices. Since the products of the Christoffel symbols would necessarily involve squares of the first derivative of

$h_{\mu\nu}$, they must also be neglected for consistency. We are then only left with the terms of the derivatives of the Christoffel symbols, which will yield the second derivatives of $h_{\mu\nu}$. Writing the curvature tensor with lower indices only, we get

$$R_{\mu\nu\rho\pi} = h_{\mu\rho,\nu\pi} + h_{\nu\pi,\mu\rho} - h_{\mu\pi,\nu\rho} - h_{\nu\pi,\mu\rho}. \tag{9}$$

One can now contract the curvature tensor to get the Ricci tensor and hence the Ricci scalar and thence the Einstein tensor. The final result is

$$\varepsilon_{\mu\nu} = \frac{1}{2}[\hat{a}\quad \Box h_{\mu\nu} + \eta^{\alpha\beta}(h_{\mu\alpha,\nu\beta} + h_{\nu\alpha,\mu\beta}) - h_{,\mu\nu} + (\Box h - h_{\alpha\beta,\alpha\beta})\eta_{\mu\nu}], \tag{10}$$

where $h = h^{\mu}_{\mu}$. The w$h_{\mu\nu}$ leads to the expectation that we can obtain the wave equation for $h_{\mu\nu}$. Remember that for the electromagnetic field we get the wave equation for the 4-vector potential only if we select the Lorentz gauge. As such, we would expect that there would be some gauge choice that would yield the required wave equation.

We know what the gauge freedom means in electromagnetism. We can add the gradient of a scalar quantity to the 4-vector potential without changing the electromagnetic field itself. What is the analogous gauge freedom for gravitation? Here the principle of general covariance plays a role. We know that changing the coordinates (including a change of frame of reference) should leave all physical laws unaltered. (Of course, we have limited ourselves to the gravitational field and there may be some doubts whether they would hold in all coordinate systems in the presence of other fields but it is taken for granted that there will be no problems in this regard.) Since these transformations obviously form a group the gauge group will be the group of all coordinate transformations in 4-dimensions. Locally we would only need infinitesimal transformations and so the Lie group would be $GL(4)$ and the corresponding Lie algebra $gl(4)$. The choice of gauge that works is obtained by defining $\phi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$. We now get the linearized field equations

$$\Box\phi_{\mu\nu} + \eta^{\alpha\beta}(\phi_{\mu\alpha,\nu\beta} + \phi_{\nu\alpha,\mu\beta}) - \phi_{\alpha\beta,\alpha\beta})\eta_{\mu\nu} = 2\kappa T_{\mu\nu}. \tag{11}$$

The gauge choice is now obvious. Writing the new potential in mixed form we take $\phi^{\alpha}_{\mu,\alpha} = 0$ to get rid of the extra three terms, so that (Hussain and Qadir, 2007) becomes the wave equation with source. *This* is the analogue of the Lorentz gauge condition, which takes thedivergence of the vector potential to be zero.

Here it is the divergence of this tensor potential. The question remains of how to achieve this gauge choice.The question now arises of how to implement this gauge choice. Since it is to be an infinitesimal transformation, we follow the procedure adopted by (Lie, 1967), to define *point transformations* while maintaining the symmetry of the manifold. To the original position position vector he added a position dependent function. (For his purposes he also separated out “independent" and “dependant” variables, but we will treat them all as equally dependent variablesthat for a curve will be given parametrically by a single parameter, a surface by two, a hypersurface by three and for the whole space will become independent variables.) Thus,

$$x^{\mu} \to \tilde{x}^{\mu} = x^{\mu} + \xi^{\mu}(x^{\nu}) . \qquad (12)$$

Since the metric must remain invariant under the transformation,

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = \tilde{g}_{\mu\nu} d\tilde{x}^{\mu} d\tilde{x}^{\nu} . \qquad (13)$$

Expanding the right side of this equation and using the approximation that terms quadratic in ξ^{μ} are neglected, we get

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} . \qquad (14)$$

By solving $\Box h_{\mu\nu} = -\xi_{\mu,\nu} - \xi_{\nu,\mu}$ we can find the required expression for ξ_{μ}. This procedure will be important for us later as well.

4. The Reality of Gravitational Waves

We have found the gravitational waves in relativity *mathematically*, but what do they amount to *physically*? The problem is that the waves should be there in the absence of any source terms, just like the electromagnetic waves in the absence of charges. Of course, the charges must have been present *somewhere* to create an electric field, but not in every region of space where the waves are. Similarly, the gravitational waves should be present in the absence of any source. Only, here the source is the energy-momentum tensor. *Its absence means that there there is no energy*! But then how could the waves have any effect on any anything — how could they act on matter? This problem was addressed by Weber and Wheeler (Weber and Wheeler, 1957). Of course, it could be argued that we used the linearizing approximation. Even in the absence of a stress-energy tensor, the nonlinear terms could be re-inserted on the right side and would then ac like an effective energy-momentum tensor (often called a *pseudo-tensor*). This does not seem a very satisfactory solution. For one thing a special frame is selected and so this concept is coordinate dependent.

There is a problem in GR that if the metric tensor is time dependent, energy is not conserved. (This is because the Hamiltonian itself has then become time dependent.) We need to say this in a coordinate independent way. In one frame the metric tensor may seem time-dependent and not in another. We can ask whether it is dime-independent in *any* frame. Is there a timelike vector that could be taken as the tangent vector to the path of an observer? One then requires that moving the metric tensor along the vector would leave it invariant, or the Lie derivative of the metric tensor relative to that vector is zero, $L_k g = 0$. A vector along which the Lie derivative of the metric tensor is zero is called a *Killing vector* or an *isometry*. Thus, if there exists a timelike Killing vector energy is conserved and if not then there is not. For there to be gravitational waves we would require that the metric tensor be genuinely time-varying. *Herein lies the problem*.In the absence of any matter or energy the stress-energy tensor will be zero. In this case the Einstein tensor must be zero. By contracting it we see that the Ricci scalar is zero. Thus the Einstein field equations reduce to the requirement that the Ricci tensor be zero (in the absence of matter and the cosmological term). As we saw above, the linearized version of this equation gives a wave equation for a field directly expressible in terms of the metric coefficients, which give the gravitational potential. The problem will be that we will then have a gravitational wave with no energy. However, if the wave is to affect matter, it must impart energy to it or absorb energy from it. If it does neither, how could it be “real"?

(Weber and Wheeler, 1957) tried to answer this question by considering test particles in the path of the putative gravitational waves. If their geodesic equations show some deviation from straight line paths in spacetime, there will effectively be energy in the waves. To avoid the possibility of the linearization approximation *itself* providing the energy, they took an solution of the field equations obtained by (Einstein and Rosen, 1937) representing cylindrical gravitational waves. The result was that momentum *would* be imparted to test particles and hence the gravitational waves effectively carried energy. They obtained a first order approximation to the momentum imparted and checked, by estimating the second order correction, that the result remained robust. Later, plane waves obtained by (Bondi et al. 1957) were used by (Ehlers and Kundt, 1962) to show that they also imparted momentum and that a sphere of test particles in the path of the waves would be distorted in different directions as they moved outwards, pushed by the waves. The gravitational waves *were* real.

5. The Energy Content of Gravitational Waves

The problem of momentum and energy was not new. It is inherent in replacing Newtonian gravity by the curvature of spacetime as a particle left near a gravitating source will start moving on its own. Thus momentum is not conserved. To provide for this discrepancy a “stress-energy *pseudo*-tensor" was defined. There were different proposals for it that I will not go into here. Many of them are reviewed and referred to in (Hussain, 2010). Essentially it amounts to taking linearized gravity, the wave operator acting on the metric tensor, as the left side of the equation and putting all the nonlinear terms on the right as the source. It is obvious that this is not a tensorial break-up and this is why it was called “pseudo". However, for the Schwarzschild metric, energy “is" conserved. Obviously it is physically unacceptable to have *this* property disturbed. Hence the various different proposals. An invariant expression for the energy in gravitational waves has remained an outstanding unsolved problem in GR.For the simple type of problem in the example above, it would be easy to take the generalized four-momentum in the metric tensor and use its contraction with the unit timelike isometry (which guarantees energy conservation) to define the energy of a test particle, or even the field as a whole. However, this naive procedure will not apply for genuinely time-varying spacetimes with no timelike isometry. Giving a measure of the energy of the particle to which momentum is imparted is easy enough ($E^2 = p^2c^2 + m^2c^4$) but that does not provide a measure for the energy in the *wave* itself.

The easiest exact solution to visualise is for cylindrical gravitational waves. Of course, there is no cylindrical gravitational wave source, which would have to be of infinite axial length, and nonlinearities could make all the difference. However, perforce we have to take some approximations as there are no spherical gravitational waves in any case and physically there are no infinite plane waves either. Since we are left without exact gravitational wave solutions, we *must* use some approximations – either in the solution or in the physical situation modeled by the solution. I prefer the latter. As such, let us consider the cylindrical waves. The metric is given by:

$$ds^2 = e^{2(\gamma-\psi)}\left(c^2dt^2 - d\rho^2\right) - e^{-2\psi}\rho^2 d\phi^2 - e^{2\psi}dz^2$$

$$\psi(t,\rho) = A[J_0(x)cos\omega t + N_0(x)sin\omega t];$$

$$\gamma(t,\rho) = \frac{1}{2}A^2 x\{J_0(x)J_0^{'}(x) + N_0(x)N_0^{'}(x)$$

$$+x[J_0(x)^2+N_0(x)^2+J_0'(x)^2+N_0'(x)^2]$$

$$+[J_0(x)J_0'(x)-N_0(x)N_0'(x)]\cos 2\omega t$$

$$+[J_0(x)N_0'(x)+J_0'(x)N_0(x)]\sin 2\omega t\}-\frac{2}{\pi}A^2\omega t\,, \qquad (15)$$

where J_0 and N_0 are the Bessel and Neumann functions respectively and $x=\omega\rho/c$.

With Sharif, I tried to address this issue by extending a formalism developed by Swadesh Mahajan, Jawaid Quamar, Prashanat Valanju and me over a period of a few years (Mahajan, 1981), (Qadir, 1986), (Qadir and J. Quamar, 1983), (Qadir and Quamar, 1986) and (Quamar, 1984). The essential idea is to reintroduce forces into GR by an operational procedure that could, in principle, provide a measure for the maximal relativistic tidal force and then defining the relativistic gravitational force as the quantity whose directional derivative along the direction of the tidal force is the tidal force. One can then define the relativistic gravitational scalar potential as the quantity whose gradient is the relativistic gravitational force. The original formalism was based on the assumption of stationarity of the spacetime, i.e. that there is no time-dependence. Sharif and I provided for the time-dependence by having the reading calibrated at one instant and then the time variation at the same place would give a measure for the time-dependent gravitational force (Qadir and Sharif, 1992) and (Sharif, 1991). (This corresponds to the choice of a particular frame of reference, which happens to be a specific Fermi-Walker frame (Qadir and Zafarullah, 1996), that is falling freely from rest at infinity. In this frame the observer sees a Minkowski space, which is the appropriate one to think in terms of for the generalization of the Newtonian viewpoint. Though GR says that there is no preferred frame, it does not forbid its use, but merely says that the laws of Physics will be simplle when stated invariantly.For particular purposes particular frames will continue to be used.) Using this formalism we were able to give a closed form expression for the momentum imparted to test particles by gravitational waves in general and cylindrical waves in particular (Qadir and Sharif, 1992)which gave the Weber-Wheeler (Weber and Wheeler, 1957) expression to first order. However, this did not give the energy *in the field*.

With (Kara et al., 1994) again tried to address the problem by using Nail Ibragimov’s definition of “approximate symmetries" (Ibragimov, 1985), but this did not give anything more than one already knew. Then, using second approximate symmetries with Ibrar Hussain and Fazal Mahomed (Hussain, 2010), (Hussain et al., 2007), (Hussain et al, 2009), (Hussain and Qadir, 2007) and(Hussain and Qadir, 2012) we managed to get a definition that passed muster. The first attempt had taken the simple point that one loses basic momentum and spin conservation in the Schwarzschild metric. Assuming that the gravitational field is weak, though the symmetries are lost they hold *approximately*. By taking the definition of approximate symmetries one can obtain a measure for them. The second one tried using the next order of approximation and we found that for this to hold we needed to re-scale the energy. The same re-scaling should then hold for the energy of the gravitational waves! Whereas the former measure depended on taking a on-zero perturbation, the re-scaling would hold even in the limit of the perturbation becoming zero. (This result is reminiscent of D'Alembert'sprinciple of virtual work, where one gets thee force of tension even in the limit of pure Statics.) The re-scaling amounts to a damping of the waves due to their nonlinear self-interaction. One can then try to assess how much this re-scaling might modify the energy that would be seen on Earth for gravitational waves that would have had much greater energy when produced. This was done assuming that the original calculation was based on a first order correction to the linear (Qadir, 2012). It turns out that the actual calculations are obtained using "numerical relativity", which goes on to higher orders. The assessment would need to be modified in light of the actual procedures used for calculation.

6. Attempts to Observe Gravitational Waves

In 1974 a binary pulsar (whose companion is also a neutron star) was observed by (Hulse and Taylor, 1975). Since the pulsar is an extremely accurate clock and the binary system provides a "vernier" reading, as it were, and because the orbit of the pair is highly eccentric, it is an ideal laboratory for GR effects. Further, since many of the effects are cumulative, observations over time can provide a higher accuracy than is even provided in the test of quantum electrodynamics. Using the parameterized post-Newtonian approximation it has been found that GR is the only theory of gravity to fit the data precisely. Those theories that introduce additional parameters have to tune those parameters sufficiently to make them indistinguishable from GR. In particular, GR predicts the rate of slowing down of the pair due to the emission of energy in the form of gravitational radiation. The observations over the last forty years follow the dotted line provide by GR exactly. As such there is very strong indirect evidence of the existence of gravitational waves. Nevertheless, direct observations would be the clincher. More recently, there is a claim to have effectively seen the ripples of gravitational waves in the data for the CMBR at the time of the supposed "inflation" that the Universe underwent.

There is an interesting aside to the above story of Hulse and Taylor, who jointly received the 1993 Nobel Prize for the discovery. The discovery of the first pulsar is credited to Anthony Hewish. As it happens it was his student, Jocelyn Bell, who had found a “smudge" on her plate during her PhD work at Cambridge under the supervision of Hewish. When she took it to her Supervisor, he told her to stop wasting her time on it and to get on with the work he had assigned. In her spare time she went on following up on it and cleared up the “smudge" to show a definite regular pulse. When she now took it to her Supervisor he got interested in it and they published three papers in the journal *Nature*. The pulses were then identified as coming from the Crab nebula and the model of a neutron star was accepted. For the discovery *Hewish* was nominated for the Prize and did not even mention that Bell should be a co-recipient! In the case of Hulse and Taylor, the former had dropped out of the academic scene and the latter is the one who followed up on the discovery for a long time afterwards. When Taylor was nominated, he immediately named his ex-student, Hulse, as a co-recipient. I have heard many people animadvert against Hewish and compare him unfavourably with Taylor.Having proved that gravitational waves are real, Joe Weber, who was originally an electrical engineer, set about trying to formulate how the waves could be actually detected. He first set up the formalism by considering a quadrupole spring detector responding to the waves as the idealized experiment. Next, he modeled a large cylindrical bar as a coherent set of quadrupoles and deduced the responses to the different modes that the waves could have in them. The total number of independent components in the potential, since it is a symmetric 4×4 matrix, is ten. However, not all of them would be independent. In fact there is the four-fold redundancy due to the gauge freedom that reduces the number of independent components to six. One common choice is to take the “transverse traceless" gauge (Misner, 1973).For the experiment he constructed a nested sequence of aluminium cylinders with a total length of about $2m$ and a diameter of about $1m$. The vibrations were to be detected by piezo-electric crystals for adequate sensitivity. In 1968, at a conference in London where I was present, (the first in my career), he claimed that he got readings once every 24 hours from a direction when the laboratory containing his bar was facing the galactic centre. This claim was published (Weber, 1968). A straightforward objection was raised that since gravitational waves interact so weakly with matter, they would simply go through the Earth and so it would be as likely to detect the waves when the laboratory was directly behind the Earth and so the period should be 12 hours. In the next conference I heard him speak he claimed to find the waves with a 12 hour period and not with the earlier 24 hour period. Another objection was that there could be spurious readings due to other sources that did not adequately get screened out by the suspension and the vibration damping precautions. He set up two and later three detectors widely spaced (a few hundred kilometers apart) in an equilateral triangle, with a sensitive seismograph at the centrum of the three. He subtracted out all events that were not coincidences of all three detectors and also any readings that coincided with those of the seismograph. He claimed that he still got the detection and invited other researchers to do the same experiment. No other researchers tried for a long time and no one who later did ever got a reading.

Finally, it was pointed out that the sensitivity of Weber's room temperature detector could not be enough as thermal vibrations would swamp out any signals. This convinced most workers. However, Weber continued to argue that the detector was sensitive enough and that he had observed the signals. He later claimed that the coherent vibration of the molecules in the bar meant that the sensitivity should be multiplied by the square of the number of molecules in the detector (which would then provide the required sensitivity). It was pointed out that coherence required quantum effects and there was no theory of quantum gravity. He then claimed that he had such a theory but would present that separately.

Many workers had by then started working with liquid helium temperature bar detectors and still did not have the sensitivity required. In 1993 we held a large international symposium on experimental gravitation in Nathiagali, Pakistan, where top people in the field, including Weber were invited. Among the scientists there was Michael Bassan who had happened to be in Weber's laboratory in 1987 when Supernova 1987A went off. Though Bassan was firmly convinced that Weber's claims were wrong, he admitted that he had seen the jump in the records of the detector at the time of the supernova. The papers of these two, and others who talked on the subject, are presented in the proceedings of the symposium (Karim and Qadir, 1994). It is worth mentioning that selected papers were also published as a special issue of *Classical & Quantum Gravity* (Karim and Qadir, 1993) but the referees had rejected Weber's contribution in the special issue. At the workshop the concensus of views (which I summarized) was that bar detectors were not good enough for the purpose and laser interferometers would have to be used.

Let me now come to the laser interferometer, which is the method we currently hope to be able to see gravitational waves with. The essential idea is that since gravitational waves are changes of the metric, we would get variations of the measure of time and distance due to the passage of the waves. Assume them to be periodic for the sake of definiteness. Then we should see expansions and contractions of distances due to the waves. It was this that Weber was looking for, but at the scale of a few metres at a time (and with the triangular arrangement of detectors at a few hundredkilometres apart, it remained oscillations of the same bar size three times over. The oscillations are less than usual thermal fluctuations in a bar of the same size at room temperature. *This* is why people would not accept Weber’s theory. Scale up the device to 4 kilometers and you gain a factor of 2,000. Further, remove the matter content so that thermal vibrations become irrelevant and just measure the distance by light rays that go across the distance and the limitation on the least measurable value is removed. For precision, use monochromatic light. As you might have seen, I have just described a $4km$ laser in a hard vacuum.

One might as well, then, use a Michelson interferometer to see fringe shifts with the large base line. This was proposed by Kip Thorne and Ronald Drever in 1992 and started operation in 2002 under the name of *Laser Interferometer Gravitational Observatory* (LIGO). So far no gravitational waves have been detected by it. While the original experiment was planned to terminate by 2010, its later phases continue and there is currently an “enhanced LIGO" in operation. To be able to study the Physics and Technology involved at a smaller scale, various “toy experiments" were set up around the world. There was one in Japan that had $400m$ arms. In 1993 I had proposed that a $2km$ arm one be set up in Pakistan with help by the Japanese through JAICA (with encouragement from JAICA). The proposal was later presented at a conference of which the proceedings were published much later (Farooqui et al., 1998). This proposal passed “a concept clearance" meeting with the Ministries of Education and Science & Technology, by late 1994. However, it then ran into a hurdle with neither Ministry wanting to take responsibility for the project, while each insisted that they were to be involved. As such, we “fell between two stools". (I am sure that this is one of the many instances that Pakistan has failed to take up a major project simply due to bureaucratic procedures and wrangling).

(Bruno Bertotti and Bernard Carr, 1989) had proposed tracking spacecraft by Doppler shift to see gravitational waves. The big advantage of this idea was that one had no matter involved and frequency shifts and time measurements are much easier to see than changes of distance. The accuracy of atomic clocks at that time was already at $:10^{-18}s$. Of course, this idea was long beore its time. There were no space borne observatories and regular experiments being carried out. What experiments were being performed were in Earth orbit or at most some little thing done on a lunar flight. The idea was taken taken up by the French and Italians under the under the name VIRGO and the Americans under the name LAGOS (for Large Gravity Observatory in Space), in the late 1980s. At that time it was still too early and nothing much came of LAGOS. However, VIRGO continued and was started in 2003 and completed in 2007. It has not, so far, managed to detect gravitational waves. A much more ambitious proposal was made by the European Space Agency, entitled LISA (Laser Interferometer Space Antenna) that won support from NASA. In 2011 NASA had to renege on their promised support due to shortage of funds to be approved by the American Congress. The original idea was to have a laser sending beams to mirrors five million kilometers away at 60^o to the laser and each other, so that they form an equilateral triangle (which would have stability). After NASA left, it was decided to have concept proving project with the distance reduced by a factor of five. The LISA Pathfinder is to be launched in September 2015. The final project is now planned for 2034.As an aside, in 1980 Asif Mufti and I had proposed a means of looking for relic gravitational waves from the Big Bang by timing discrepancies by atomic clocks (Qadir and Mufti, 1980). Unfortunately, one required a million times the accuracy in the clocksand needed that it should be possible to place them about 5 to 10 cm apart. This is obviously far ahead of its time and I do not know if it will ever be possible to achieve it.

7. Gravitational Wave Sources

As mentioned earlier, the biggest problem in the detection of gravitational waves is the extremely weak coupling between gravity and matter. This claim may seem paradoxical, since the force that we most easily see in Nature *is* the gravitational force. The reason for it is that whereas the strong andweak nuclear forces are much stronger they are short range forces that will not be felt beyond $\sim 10^{-13} cm$ and, since matter is generally electromagnetically neutral we see only a little bit of a remnant of the electromagnetic as some individual charges or higher multipole fields, but for the bulk of the matter the major part of the force has cancelled out. However, the gravitational force is additive for all matter. Thus, though the coupling is about forty orders of magnitude less than the electromagnetic coupling, even for relatively low mass objects like the Moon, the gravitational force is all that is normally seen. One might have thought that a supernova explosion would easily provide a strong enough source but even in that case but the otehr complication is that there are no spherical gravitational waves. Hence that mode in the waves, which should have dominated, is removed. We are only left with higher multipoles of the radiation field. In this section when I talk about being detectable, I am referring to the standard expectations and not to my prediction that the waves will be further damped.

Let us be more concrete. For plane gravitational waves, in the transverse traceless gauge (mentioned earlier) one can choose the perturbation $h_{\mu\nu}$ to have four non-zero components: $(h_{11} = -h_{22} = h_+; h_{12} = h_{21} = h_\times) \times e^{i(kz-\omega t)}$ (which are only two independent constants). The generic perturbation, h, gives a "strain" amplitude $(\Delta L/L)$. To get an idea of the magnitude of the h, if we take a take a dumbbell consisting of two one ton lead balls spinning about their common centre of mass at a separation of $2m$, frequency $1kHz$ and a distance of 300 km, $h : 10^{-38}$! It is, thus, obvious that there is no hope for a laboratory test of the prediction. Instead, take the type of objects that we know exist, binary neutron stars. We already know that the Hulse-Taylor binary pulsar emits gravitational radiation, because of the slowing down of binary period. However, they are too weak to be seen on Earth with present (or slightly projected) technology. Instead, consider the binary pair in the extreme case when they are just about to merge. An example is given (Saulson, 1994) of a pair of mass $1.4M_\odot$ (the Chandrasekhar limit for hot stars) in a circular orbit of $20km$ and a period of $0.25ms$ seen at a distance of $15Mpc$ (where $1Mpc : 10^{20} km$) is

$$h \approx \frac{10^{-21}}{(r/15Mpc)}. \tag{16}$$

This is the type of amplitude that LIGO and Virgo could hope to detect. Of course, one needs to have the merger in the process of occurring. As is clear from Eq. (16), if a merger occurred within our galaxy the amplitude would increase by a factor of a thousand. However, such mergers are expected to be very rare and we have no real expectation of seeing one in our own galaxy.

An alternative source to look for would be in supernovae events. These are, again, transient phenomena. The strain amplitude in this case is given by

$$h : 6\times10^{-21}(\frac{E}{10^{-7}M_\odot c^2})^{1/2}(\frac{1ms}{T})(\frac{1khz}{f})(\frac{10kpc}{r}), \tag{17}$$

where E is the energy of the explosion, T the time interval over which the explosion occurred, f the frequency of the radiation and r the distance of the supernova from the Earth. The recent supernova almost within our galaxy (in the Larger Magellanic Cloud to be precise), SN1987A, was $: 50kpc$ away and should have been detectable, but neither VIRGO nor LIGO were operational at the time. The Larger and Smaller Magellanic clouds are dwarf galaxies in orbit about our Milky Way. Theclosest full galaxy is M31, the Andromeda galaxy at $: 780kpc$. A merger there should also be detectable, especially with the upgraded Virgo. R ecall that the problem is the normal sphericity of supernovae, now reasonably well understood on the basis of the data from SN1987A. If the asphericity were much larger, due to differential rotation, we could hope for much stronger gravitational waves to emerge. For such supernovae (if they occur) the expectation is that waves would be seen even at about $10Mpc$.The binary pulsar has the big advantage of being a continuous source for gravitational waves. While one has to catch the transient source in the act of emitting, the continuous source gives a much larger “window" of time. A single neutron star could be a viable source if it was a pulsar that is “spinning down" adequately fast. Now one would be looking for pulsars within our own galaxy, of which there are numerous examples. However, few of them would be spinning down very fast. The strain amplitude here would be

$$h \approx 2.5\times 10^{-25}(\frac{1kpc}{r})\sqrt{(\frac{1khz}{f})(\frac{-\dot{f}}{10^{-10}\,Hz/s})(\frac{I_z}{I_o})}\ , \tag{18}$$

where f f is the pulsar frequency, I_Z is the diagonal Z component of the moment of inertia tensor of the pulsar and I_O is a nominal quadrupole moment of the pulsar with a value of $10^{38} kg.m^2$. While it is a continuous source one has to catch the pulsar at the age when it is slowing down fast enough. The strain amplitude here is

$$h \approx 2.2\times 10^{-24}(\frac{1kpc}{r})\sqrt{(\frac{1000yr}{\tau})(\frac{I_z}{I_o})}\ , \tag{19}$$

where τ is a quarter of the the negative logarithmic derivative of the frequency. One could also look for stochastic sources that, for the present purpose are continuous. Of course, here one is faced with much greater ambiguity as it depends on the nature of the stochastic source. I am not giving a discussion of this here. Details can be obtained from the open arXiv (Riles, 2012).

Let us hope that some of the readers of this article will be around to see the direct observation of gravitational waves. Once they are detected, they will open up anew window on to the Universe which will see right through matter but whose distortions will show pictures of the intervening matter in an X-ray, as it were. It might be recalled that I have a prediction that the observed waves will have a much lower energy than expected, making them more difficult to observe than is predicted by numerical relativity calculations (Qadir, 2012).

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